

MODULE-2

- **Determination of Physico-Mechanical properties of Rock and their Determination**
- **Rheological Properties of Rock and their Determination**

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Module 2 PHYSICAL AND MECHANICAL PROPERTIES OF ROCK

(i) The performance of a rock, under a particular condition, depends upon the physical and mechanical properties of the rock.

(ii) The physical properties also known as index property helps in describing the material of the rock and in classifying the rock.

(iii) The Mechanical property also known as strength property helps in describing the performance of the rock, when subjected to a particular loading system.

Note

same

physico-Mechanical properties of a rock may vary drastically from place to place and even from time to time in a same geological formation.

Physical property

1. Density (Kg/m^3)
2. Specific Gravity (Unitless)
3. Porosity (Unitless)
4. Void Ratio
5. Moisture Content
6. permeability
7. Swelling property

Mechanical Property

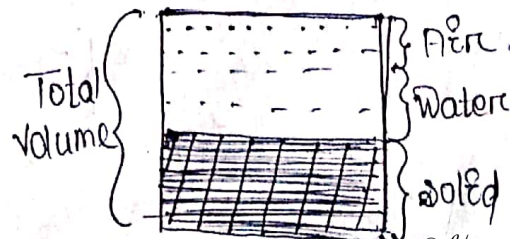
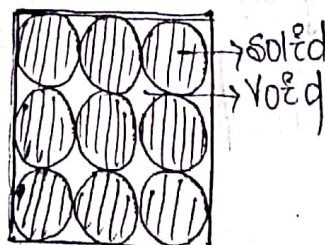
1. Modulus of Elasticity (E)
2. Uniaxial Compressive strength (MPa)
3. Tensile strength (MPa)
4. shear strength
5. point load strength
6. Poisson Ratio

* Dry density (ρ_d) = $\frac{\text{Mass of the solid}}{\text{Total Volume}} = \frac{\text{Kg}}{\text{m}^3}$

* Unit weight of solid = $\frac{\text{Mass of dry solid}}{\text{True Volume}}$

True Volume signifies the grains without pores and fissures.

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* Bulk unit Weight = $\frac{\text{Mass of a dry solid Rock}}{\text{Volume Enclosing pores and (Pores and)}} \rho_{\text{bulk}}$

* Dry density means mass per unit Volume When the rock is dry & void contains only air.

* Bulk density Means mass per unit Volume in normal condition & void contain some liquid and some air.

* Wet density Means mass per unit Volume When (or saturated) the rock mass is fully saturated

Relation between dry density and Bulk density

$$\rho_d = \frac{\gamma}{1+m}$$

Where

ρ_d = dry density

γ = Bulk density

m = Moisture content of the sample

Moisture Content (m)

(i) Moisture Content of a rock sample is defined as the ratio of weight of water in void to the weight of solid of the sample.

(ii) $m = \frac{W_w}{W_s}$ or $m = \frac{\text{Mass wet} - \text{Mass dry}}{\text{Mass dry}}$

$$V_{\text{total}} = V_{\text{solid}} + V_{\text{water}} + V_{\text{air}}$$

When the rock is dry

$$V_{\text{total}} = V_{\text{solid}} + V_{\text{air}}$$

Void Ratio (e)

$$e = \frac{V_{\text{void}}}{\text{Volume of solid}} \Rightarrow \left[e = \frac{V_v}{V_s} \right]$$

Porosity

Porosity (η) = $\frac{\text{Volume of void}}{\text{Total Volume}} = \frac{V_v}{V_s} \Rightarrow \left[\eta = \frac{V_v}{V} \right] \times 100$

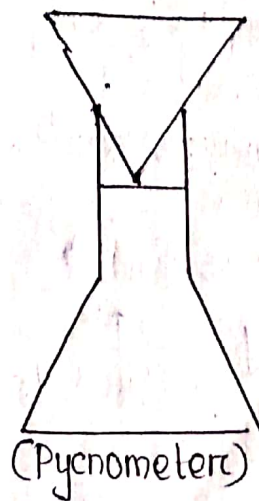
Relation between porosity and void Ratio

$$\eta = \frac{V_v}{V} = \frac{V_v}{V_s + \frac{V_v}{e}} \Rightarrow \left[\eta = \frac{e}{e+1} \right]$$

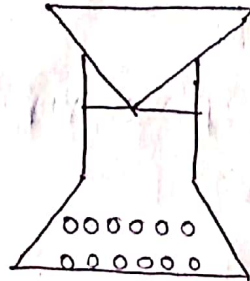
Specific Gravity (G)

$$G = \frac{f_s}{f_w} \Rightarrow G = \frac{M_s}{f_w \times V_s}$$

Where M_s - Mass of solid
 f_w - density of water
 V_s - Volume of solid



Empty
 w_1



Dry Coal
 w_2



Water +
 dry coal
 w_3



Water
 w_4

w_1 = Weight of the pycnometer

w_2 = Weight of the pycnometer + dry coal

w_3 = Weight of the pycnometer + dry coal + water

w_4 = Weight of the pycnometer + water

$$\text{Mass of the solid} = w_2 - w_1$$

$$\text{Volume of the solid} = \frac{(w_4 - w_1) - (w_3 - w_2)}{f_w}$$

Q Dry sandstone sample of dia = 22mm and height 35mm. Weight 30 gm. The sample is crushed. Find the specific gravity having the following data.
 $w_3 = 91$ gm, $w_4 = 72$ gm and also find the porosity of the sample.

Ans Given, $d = 22$ mm, $h = 35$ mm

$w_2 - w_1 = 30$ gm = Mass of the solid

$w_3 = 91$ gm, $w_4 = 72$ gm

We know,

$$\text{Volume of the sand} = \frac{(W_4 - W_1) - (W_3 - W_2)}{\rho_w}$$

$$= \frac{W_4 - W_3 + W_2 - W_1}{\rho_w}$$

$$= \frac{(72.91 + 30) \times 10^{-3}}{1000}$$

$$= 11 \times 10^{-6} \text{ m}^3$$

$$\text{Now, specific gravity} = \frac{M_s}{\rho_w \times V_s} = \frac{30 \times 10^{-3}}{1000 \times 10^{-6} \times 11}$$

$$\Rightarrow \boxed{G = 2.72}$$

$$\text{Again } V_T = V_s + V_v$$

$$\Rightarrow V_v = \frac{\pi}{4} \times (22 \times 10^{-3})^2 \times 35 \times 10^{-3}$$

$$= 1.33 \times 10^{-5} \text{ m}^3$$

$$V_s = 11 \times 10^{-6} \text{ m}^3$$

$$\therefore V_v = V_T - V_s = 1.33 \times 10^{-5} - 11 \times 10^{-6} =$$

$$\Rightarrow \boxed{V_v = 2.3 \times 10^{-6} \text{ m}^3}$$

$$\text{porosity} = \frac{V_v}{V_T} = \frac{2.3 \times 10^{-6}}{1.33 \times 10^{-5}} = 0.172$$

Permeability

(i) permeability test is based on the Darcy's law.

(ii) He proposed that for Laminar and steady flow the discharge velocity (v) from the end discharge point is proportional to the hydraulic gradient. Where,

$$v \propto \frac{\Delta h}{\Delta L}$$

q = Rate of water flow (cm^3/sec)

$$\Rightarrow v = K \frac{\Delta h}{\Delta L}$$

K = hydraulic conductivity

A = Area of cross-section of sample

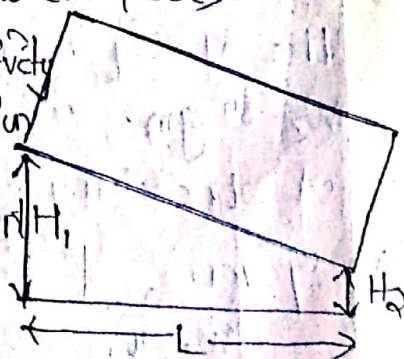
$$\Rightarrow v = KI$$

I = hydraulic gradient H_1

$$\Rightarrow \boxed{q = KAI}$$

H = Head of water (cm)

L = length of specimen



Again $q = KAI = \frac{Q}{t}$
 Where Q = Volume of Water Collected
 t = Duration of Water Collected
 K = Hydraulic Conductivity or Coefficient of permeability

There are two methods to find the value of K

1. Constant head Method

We know,

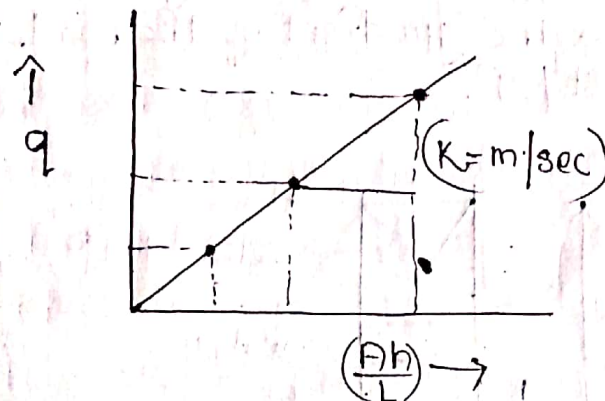
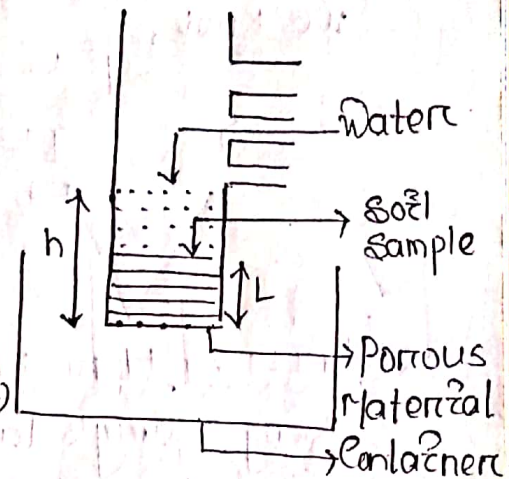
$$q = KAI$$

$$\Rightarrow q = KA \frac{h}{L}$$

$$\Rightarrow q = K \left(\frac{Ah}{L} \right) \rightarrow (i)$$

Again $y = mx \rightarrow (ii)$

Comparing equation (i) and (ii)
 We get



Q. A Constant head permeameter has a sample of medium grain sand 15 cm in length and 25 cm² cross-sectional area with a head of 7 cm. A total of 110 ml of water is collected in 12 minutes. Find the hydraulic conductivity?

Ans Given $L = 15 \text{ cm} = 15 \times 10^{-2} \text{ m}$
 $A = 25 \text{ cm}^2 = 25 \times 10^{-4} \text{ m}^2$
 $H = 7 \text{ cm} = 7 \times 10^{-2} \text{ m}$
 $Q = 110 \text{ ml} / 12 \text{ min}$

Now, $q = 110 \text{ ml} / 12 \text{ min}$

$\Rightarrow 12 \text{ min} = 110 \text{ ml}$

$\Rightarrow 1 \text{ min} = \frac{110 \times 10^{-3}}{12} \text{ l}$

$\Rightarrow 1 \text{ sec} = \frac{110 \times 10^{-3}}{12 \times 60} = 0.0001527$

$\therefore q = 0.0001527 \text{ l/sec}$

$= 0.0001527 \times 10^{-3} \text{ m}^3/\text{sec}$

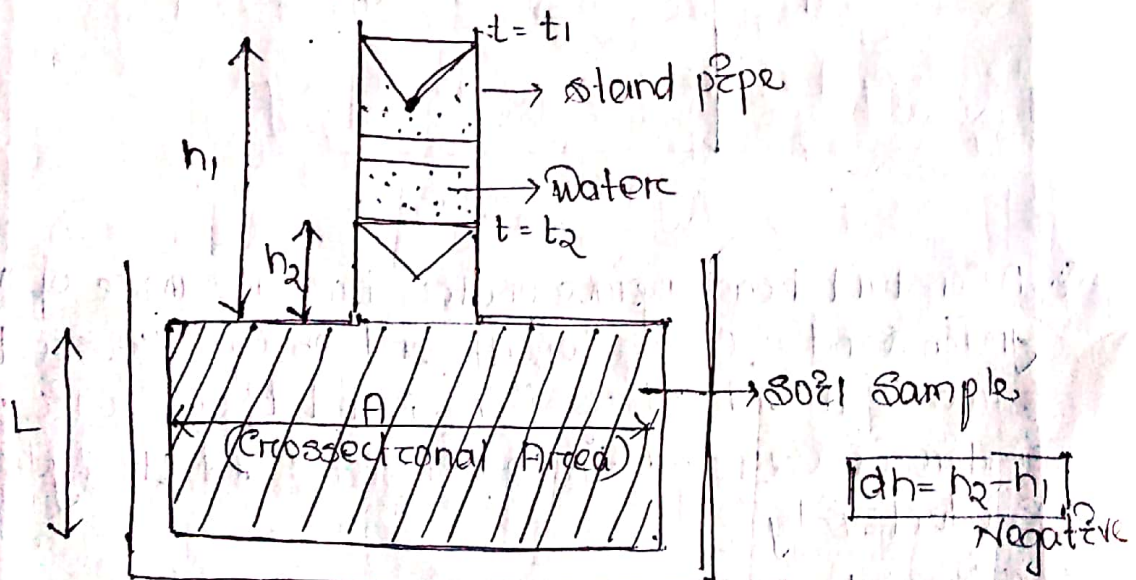
We know,

$q = KA \frac{h}{L}$

$\Rightarrow K = \frac{qL}{Ah} = \frac{0.0001527 \times 10^{-3} \times 15 \times 10^{-2}}{25 \times 10^{-4} \times 7 \times 10^{-2}}$
 $= 0.00013 \text{ m/sec (Ans)}$

2. FALLING HEAD METHOD

Using Darcy's law, we can state that quantity flowing through the sample length L at time t is related to the gradient of the head with respect to time dt/dh .



Now, $q = KAV$

$\Rightarrow q = Ap \times v$

$\Rightarrow K A \frac{dh}{dt} = Ap \times v$

$\Rightarrow K \Rightarrow dt = K \times \frac{A}{Ap} \times dh$
 $\Rightarrow \int_{t_1}^{t_2} dt = \frac{ApL}{AK} \int_{h_1}^{h_2} \frac{dh}{h}$

$$\Rightarrow (t_2 - t_1) = \frac{A p L}{A K} \ln(h_2 | h_1)$$

$$\Rightarrow K = \frac{A p L}{(t_2 - t_1) A} \ln \frac{h_1}{h_2}$$

$$\Rightarrow K = 1.2303 \frac{A p L}{(t_2 - t_1) A} \log \frac{h_1}{h_2}$$

Q. A falling head permeameter containing a fine sand has a falling head tube (standing tube) of dia 4 cm. A sample dia 12 cm and flow length is 15 cm. The initial head is 5 cm and it fall to 0.5 cm. With time 528 min. Find K?

Ans $d_p = 4 \text{ cm}$

$$A_p = \frac{\pi}{4} \times (4 \times 10^{-2})^2$$

$$d = 12 \text{ cm}$$

$$A = \frac{\pi}{4} \times (12 \times 10^{-2})^2$$

$$L = 15 \text{ cm} = 15 \times 10^{-2}$$

$$h_1 = 5 \text{ cm} = 5 \times 10^{-2} \text{ m}$$

$$h_2 = 0.5 \text{ cm} = 0.5 \times 10^{-2} \text{ m}$$

$$t = 528 \text{ min.}$$

$$K = \frac{A p L}{(t_2 - t_1) A} \ln \frac{h_1}{h_2}$$

$$= \frac{\frac{\pi}{4} \times (4 \times 10^{-2})^2 \times 15 \times 10^{-2}}{528 \times 60 \times \frac{\pi}{4} \times (12 \times 10^{-2})^2} \ln \left(\frac{5 \times 10^{-2}}{0.5 \times 10^{-2}} \right)$$

$$= 1.211 \times 10^{-6} \text{ m/sec}$$

Slake durability (Index I_d)

Rock sample after set of drying and wetting cycle.

A- Initial Mass of the drum + dry rock sample.

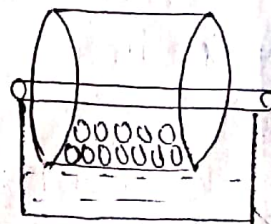
B- Mass of the drum + Mass of the rock after first cycle.

C- Mass of the drum + Mass of the rock after second cycle.

D- Mass of the drum

$$I_{d1} = \frac{B-D}{A-D} \times 100 \quad (\text{First Cycle})$$

$$I_{d2} = \frac{C-D}{A-D} \times 100 \quad (\text{Second Cycle})$$



Very high durability = 799

Id₂
798

Very low durability = < 60

< 30

Strength

(i) The ability of a material to resist an external applied load is called strength.

(ii) Rock strength depends upon the following factors.

1. Size of the rock specimen

2. Type of test

3. Duration of test

4. Loading condition

5. Cycle of loading

6. Degree of saturation

7. Confining pressure

Tensile strength
Ability of a material to resist maximum tensile stress.

Shear strength
Ability of a material to resist maximum shear stress.

(iii) Strength is of three types

1. Tensile strength

2. Compressive strength

3. Shear strength.

Compressive strength

It is again of two types

a) Uniaxial or Unconfined Compressive strength

b) Triaxial Compressive strength.

a) UNIAxIAL / UNCONFINED COMPRESSIVE STRENGTH

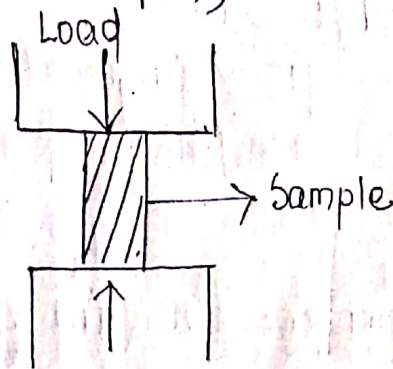
(i) Uniaxial / Unconfined Compressive strength is the strength of the rock when a load on the rock acts in one direction only.

(ii) There is no load along an axis perpendicular to the loading axis.

(iii) The uniaxial Compressive strength is the value of Compressive strength at which the specimen or sample break or fail.

(iv) This test is conducted in straight, circular cylinders having height to dia ratio (H/D = 2.0 - 3.0)

and diameter preferably not less than N_x core diameter (54mm).

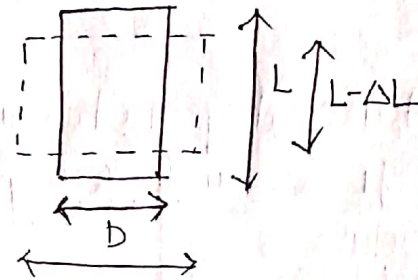


Test of UCS

- (i) Coring
- (ii) Polishing
- (iii) Testing

* Stress = $\frac{\text{Force}}{\text{Area}} \Rightarrow \sigma = \frac{N}{m^2} = \text{N/m}^2$

*



* Longitudinal strain = $\frac{\text{change in length}}{\text{Original length}} = \frac{-\Delta L}{L}$

* Lateral strain = $\frac{\text{change in diameter}}{\text{Original diameter}} = \frac{\Delta D}{D}$

* Poisson's Ratio = $-\frac{\text{Lateral strain}}{\text{Longitudinal strain}}$
 $= -\frac{\frac{\Delta D}{D}}{\frac{-\Delta L}{L}}$

Poisson's Ratio = (0 - 0.5)

Hooke's law

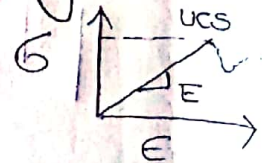
Within elastic limit, stress is directly proportional to strain.

$$\sigma \propto \epsilon$$

$$\Rightarrow \sigma = E \epsilon$$

$$\Rightarrow \boxed{E = \frac{\sigma}{\epsilon}}$$

E = Young's Modulus or Modulus of elasticity.



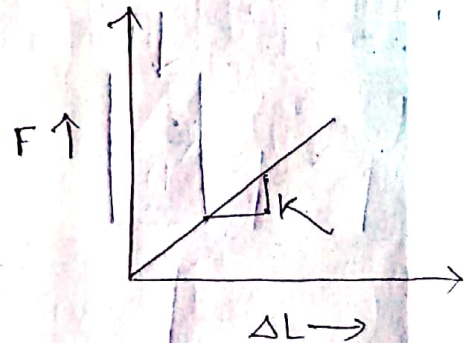
Triaxial Compressive strength

When a rock mass is subjected to an all around pressure and further it is subjected to an additional vertical pressure than the strength exhibited by the rock mass is known as triaxial compressive strength.

Stiffness (K)

$$K = \frac{F}{\Delta L}$$

Relationship between stiffness and Young's Modulus



$$E = \frac{\sigma}{\epsilon} = \frac{\frac{F}{A}}{\frac{\Delta L}{L}} = \frac{F}{A} \times \frac{L}{\Delta L} = \frac{F}{\Delta L} \times \frac{L}{A} = \frac{KL}{A}$$

$$\Rightarrow \boxed{E = \frac{KL}{A}}$$

$$\boxed{\text{Resilience} = \frac{\sigma^2}{2E} \times \text{Volume}}$$

Energy

$$\boxed{E = \frac{1}{2} \times \Delta L \times F} \rightarrow \text{①}$$

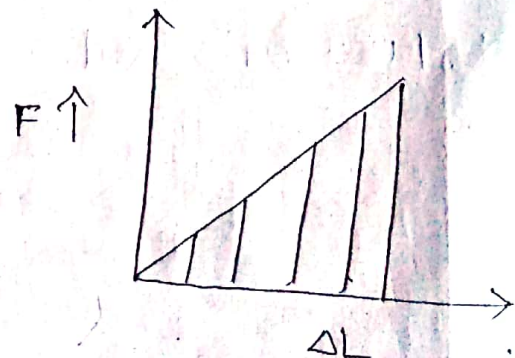
= potential energy / strain energy

$$\Rightarrow E = \frac{1}{2} \times \sigma \times \epsilon \times A \times L$$

We know, $\sigma = E \epsilon$

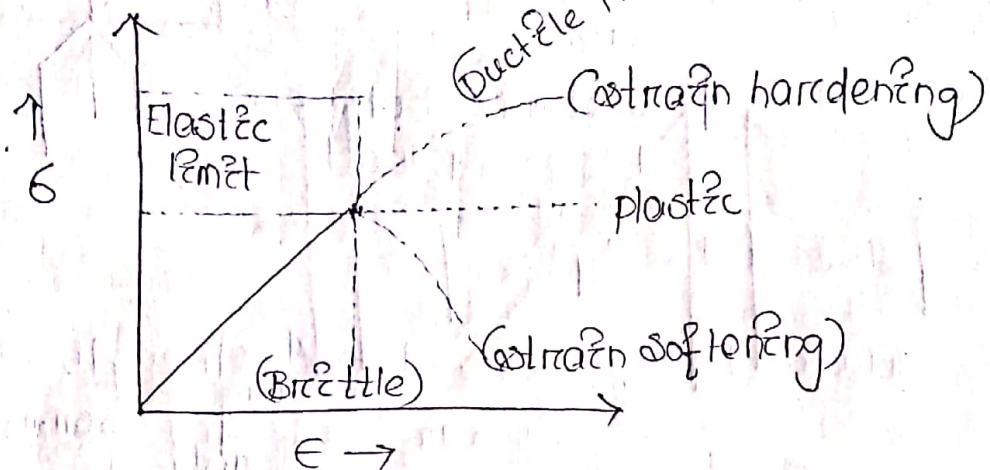
$$= \frac{F}{A} \times \frac{L}{\Delta L}$$

$$\Rightarrow \boxed{\Delta L = \frac{FL}{AE}}$$



putting the value of ΔL in eqn (i),

$$E = \frac{1}{2} \times \frac{FL}{AE} \times F \Rightarrow \boxed{E = \frac{F^2 L}{2AE}}$$

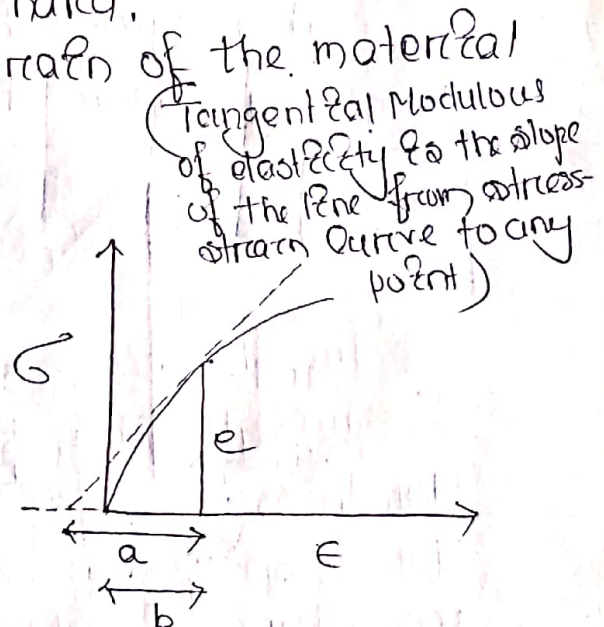


* Strain hardening = After confinement the material becomes too hard.

* Strain softening = Strain of the material releases slowly.

* Tangential Modulus of elasticity = $\frac{\sigma}{\alpha}$

* Secant Modulus of elasticity = $\frac{\sigma}{b}$



TENSILE STRENGTH

* Indirect tensile strength test / Brazilian tensile strength test.

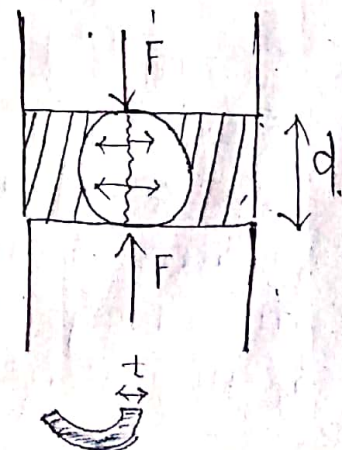
$$\frac{L}{D} = 0.5 = \frac{1}{2} = 1:2$$

$$\sigma_t = \frac{2F}{\pi dt}$$

F = load acting on the sample

d = dia of sample

t = thickness of sample.

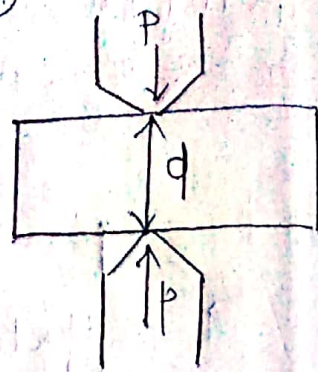


Point load strength Index (I)

$$I = \frac{P}{d^2}$$

Where P = load at rupture/
break

d = Distance between
two point load.



$$\sigma_{ucs} = 24 \times I_{d=50mm}$$

Where $I_{d=50mm}$ = point load strength index
of sample of ϕ 50mm

* Relationship between computed ^{Core} ultimate compressive strength and having of sample having $\frac{L}{D}$ ratio equals to 1 and sample having length greater than ϕ is given by

$$\sigma_0 = \frac{\sigma_c}{0.778 + 0.222\left(\frac{P}{L}\right)}$$

σ_0 = strength when $\frac{L}{D} = 1$

σ_c = strength when $\frac{P}{L} > 1$ $\frac{L}{D} > 1$

Hardness

Hardness of a rock is defined as the resistance to abrasion.

Punch shear strength

$$\tau = \frac{P}{\pi d t}$$

Swelling

Swelling of a rock is defined as the increase in volume of the rock mass after absorbing water or remain in contact with water for long period.



(punch shear test)

MODULE-02

The most important properties for application of elastic theories are

- a) Modulus of elasticity (E)
- b) Modulus of Rigidity (G)
- c) Poisson's Ratio (γ)

MODULUS OF ELASTICITY (E)

(i) When a test specimen is loaded within the compression or tension within the elastic range then the ratio of stress to the strain in the direction of stress is known as young's modulus or Modulus of elasticity (E).

MODULUS OF RIGIDITY (G)

(i) When a test specimen is loaded in shear, then the ratio of shear stress to shear strain is known as modulus of rigidity (G).

POISSON'S RATIO (γ)

(i) The ratio of lateral strain to the longitudinal strain is called Poisson's Ratio.

Relation between E, G and γ

$$E = 2G(1 + \gamma)$$

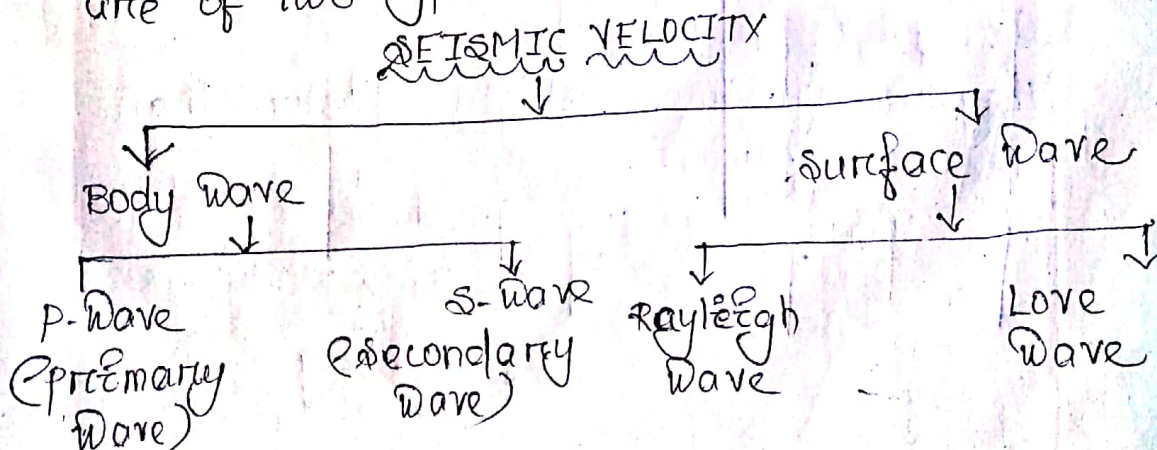
$$E = 3K(1 - 2\gamma)$$

$$E = \frac{9KG}{3K + G}$$

SEISMIC VELOCITY

(i) The seismic velocity is the propagation of elastic wave through a particular rock.

(ii) The seismic velocity of a material property are of two types.



BODY WAVES

(i) Body waves are those which pass through the interior of mass.

(ii) These are again of two types.
a) P-Waves (Primary / longitudinal waves)
b) S-Waves (Secondary / shear waves)

a) P-Waves

(i) P-Waves are longitudinal waves / are compressional waves in solid.

(ii) These waves travel twice the speed of S-waves.

b) S-Waves

(i) S-Waves are transverse or shear waves which means the ground is displaced perpendicular to the propagation of transverse.

SURFACE WAVE

(i) Surface waves are those waves which pass along the surface.

(ii) These are again of two types

a) Rayleigh wave

b) Love wave

Rayleigh wave

(i) These are also called surface wave and they travel very similar to these waves on the surface of water.

Love wave

(i) Love waves are those surface waves that cause horizontal shaking of ground.

(ii) They usually travel slightly faster than the Rayleigh wave.

$$V_p = \sqrt{\frac{4G + K}{\rho}}$$

$$V_s = \sqrt{\frac{G}{\rho}}$$

where G = shear modulus

K = Bulk modulus

ρ = density of rock